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Simulation of Technical Systems on the basis of Vector Optimization (2. with a Criterion Priority)

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Abstracts

The new methodology of modeling of the technical system (TS) which model is presented by the vector problem of mathematical programming (VPMP) is presented. The technique of an assessment and choice of necessary technical characteristics (at equivalent criteria) and determination of the TS corresponding parameters is presented in the first part. In this work theoretical justification (axiomatics), creation of mathematical model of technical system in the form of VPMP and its decision based on normalization of criteria and the principle of guaranteed result with a priority of criterion is given. The methodology of modeling is illustrated on a numerical example of the TS mod-el with a criterion priority, in the form of a vector problem of nonlinear pro-gramming realized in Matlab system.

Keywords: modeling of technical systems, vector problem of mathematical programming.

Introduction

At research and modeling of new technical objects, the systems (TS) which model is presented by the vector problem of mathematical programming (VPMP) [5, 6, 7, 9], there is a problem of an assessment of results of modeling and making optimal decisions based on them [4, 8]. When modeling the technical systems (TS) much attention, both in domestic science (Russia) [3, 11], and in foreign scientific activity in theoretical [1] and applied aspects [12, 13] is paid to a decision-making problem.

The technique of an assessment and choice of necessary technical characteristics (at equivalent criteria) and determination of the TS corresponding parameters is presented in the first part [10].

In real life there are problems of evaluation of research results TC, not only at equivalent criteria, but also at a certain importance (priority) of the criterion. This work is directed on the solution of these problems.

The purpose of this work – theoretical justification, methodologies of creation of model and mathematical modeling of technical system in the form of the vector problem of mathematical programming (VPMP) and its decision at a criterion priority.

For goal realization in work the model of technical system in the form of a vector problem of mathematical programming is presented. Theoretical justification (axiomatic) of the solution of VPMP with a priority of criterion is given and the algorithm of the solution of VPMP with the criterion priority, based on normalization of criteria and the principle of the guaranteed result is presented. The methodology of modeling is illustrated on a numerical example of the TS model, in the form of the vector problem of nonlinear programming solved with a priority of one of criteria, realized in Matlab [2] system.

Mathematical model of the technical system

The problem of a choice of optimum parameters of technical systems according to functional characteristics arises during the studying, the analysis and design of technical systems within automatic design of the TS.

In [9, 10] it is shown that the mathematical model of technical system solving as a whole a problem of a choice of the optimum design decision (a choice of the TS optimum parameters), it is possible to present in the form of a vector problem of mathematical programming:

$$Opt\ F(X) = \{max\ F_1(X) = \{max\ f_k(X),\ k = \overline{1, K_1}\}, \quad (1)$$

$$min\ F_2(X) = \{min\ f_k(X),\ k = \overline{1, K_2}\}\}, \quad (2)$$

$$G(X) \leq 0, \quad (3)$$

$x_j^{\min} \le x_j \le x_j^{\max}$, $j=\overline{1,N}$, (4)

where X - a vector of operated variables (design data of the TS), $\boldsymbol{N}$ - a set of design parameters[1]: $X=\{x_1\ x_2\ \dots\ x_N\}$ where N - number of parameters, each of which lies in the set limits:

$x_j^{\min} \le x_j \le x_j^{\max}$, $j=\overline{1,N}$, or $X^{min} \le X \le X^{max}$, (5)

$x_j^{\min}$, $x_j^{\max}$, $\forall j \in \boldsymbol{N}$ - upper and lower limits of change of a vector of the TS parameters;

$F(X)=\{f_k(X),\ k=\overline{1,K}\}$ - the vector criterion which everyone component submits the characteristic of the TS which is functionally depending on a vector of variables X, is supposed that the functions $f_k(X)$, $k=\overline{1,K}$ are differentiated and convex, a subset of the functions $F_1(X)=\{f_k(X),\ k=\overline{1,K_1}\}$ accepts the greatest possible value, and a subset of the functions $F_2(X)=\{f_k(X),\ k=\overline{1,K_2}\}$ accepts the minimum value, the Set of characteristics (criteria) $\boldsymbol{K}=\boldsymbol{K}_1 \cup \boldsymbol{K}_2$;

$G(X)=(g_1(X)\ g_2(X)\ \dots\ g_M(X))^T$ – a vector function of the restrictions imposed on functioning of the TS, $g_i(X)$, $i=\overline{1,M}$ are continuous, and (3)-(4) set of admissible points of $\boldsymbol{S}$ set by restrictions isn't empty and represents a compact:

$\boldsymbol{S}=\{X \in \boldsymbol{R}^N \mid G(X) \le 0,\ X^{min} \le X \le X^{max}\} \ne \emptyset$.

Restrictions are defined proceeding in them technological, physical and to that by similar processes and can be presented in the form of functional restrictions, for example, $f_k^{\min} \le f_k(X) \le f_k^{\max}$, $k=\overline{1,K}$.

Relations (1)-(4) form a mathematical model of the TS. It is required to find such vector of the $X^o \in \boldsymbol{S}$ parameters at which everyone a component the vector - function the $F_1(X)=\{f_k(X),\ k=\overline{1,K_1}\}$ accepts the greatest possible value, and the vector - function $F_2(X)=\{f_k(X),\ k=\overline{1,K_2}\}$ accepts a minimum value.

To solve this class VPMP in this article uses the methods based on the principle of normalization criteria and the principle of guaranteed result [4, 5, 8]. They allow you to decide when VPMP equivalent criteria and priority for a given criterion.

Result functioning technical system defined by the set $\boldsymbol{K}$ to technical characteristics of $f_k(X^o)$, $k=\overline{1,K}$, which are functionally dependent on the design parameters of the TS $X^o=\{x^o_j,\ j=\overline{1,N}\}$, together they represent a vector function:

$F(X^o) = (f_1(X^o)\ f_2(X^o)\ \dots\ f_K(X^o))^T$. (6)

In this article the main attention is paid to TS modeling according to the solution of VPMP with a priority of any criterion.

Decision vector problems with a criterion priority

Theoretical Foundations of the solution of problems of vector optimization with a criterion priority

For development of methods of the solution of problems of vector optimization with a priority of criterion we will enter definitions:

- priority of one criterion of VPMP over others criteria;
- numerical expression of a priority;
- the set priority of criterion;
- the lower (minimum) level from all criteria with a priority of one of them;
- about a subset of points, priority by criterion (Axiom 2);
- the principle of an optimality of the solution of problems of vector optimization with the set priority of one of criteria;

[1] Another way to write the vector $X=\{x_j,\ j=\overline{1,N}\}$.

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and related theorems. For more details see [4, 5, 8].

Definition 1. The criterion of $q \in \boldsymbol{K}$ in VPMP in a point of $X \in \boldsymbol{S}$ has a priority over other criteria of $k = \overline{1, K}$ relative estimate of $\lambda_q(X)$ by this criterion more or is equal relative estimates of $\lambda_k(X)$ of other criteria, i.e.:

$\lambda_q(X) \geq \lambda_k(X)$, $k = \overline{1, K}$,

and a strict priority, if at least for one criterion of $t \in \boldsymbol{K}$,

$\lambda_q(X) > \lambda_t(X)$, $t \neq q$,

and for other criteria of $\lambda_q(X) \geq \lambda_k(X)$, $k = \overline{1, K}$, $k \neq t \neq q$.

Introduction of definition of a priority of criterion in VPMP executed redefinition of early concept of a priority. If earlier in it the intuitive concept about importance of this criterion was put, now this "importance" is defined by mathematical concept: the more the relative estimate of q-th of criterion over others, the it is more important (more priority), and the highest priority in a point of an optimum of X_k^* , $\forall q \in \boldsymbol{K}$.

From definition of a priority of criterion of $q \in \boldsymbol{K}$ in VPMP follows that it is possible to reveal a set of points of $\boldsymbol{S_q} \subset \boldsymbol{S}$ which is characterized by that $\lambda_q(X) \geq \lambda_k(X)$ $\forall k \neq q$ $\forall X \in S_q$. But the answer to a question of, as far as criterion of $q \in \boldsymbol{K}$ in this or other point of a set of S_q is more priority than the others, remains open. For clarification of this question we will enter communication coefficient between couple of relative estimates of q and k which in total represent a vector:

$P^q(X) = \{p_k^q(X) \mid k = \overline{1, K}\}$ $q \in \boldsymbol{K}$ $\forall X \in S_q$.

Definition 2. In VPMP with a priority of criterion of q-th over other criteria of $k = \overline{1, K}$, for $\forall X \in S_q$, a vector of $P^q(X)$ which everyone component shows in how many time a relative estimate of $\lambda_q(X)$, $q \in \boldsymbol{K}$, is more than other relative estimates of $\lambda_k(X)$, $k = \overline{1, K}$, we will call *numerical expression of a priority of q-th of criterion over other criteria* of $k = \overline{1, K}$, i.e.

$P^q(X) = \{p_k^q(X) = \lambda_q(X)/\lambda_k(X),\ k = \overline{1, K}\}$, (7)

$p_k^q(X) \geq 1$, $\forall X \in S_q \subset S$, $k = \overline{1, K}$, $\forall q \in \boldsymbol{K}$.

Definition 3. In VPMP with a priority of criterion of $q \in \boldsymbol{K}$ for $\forall X \in \boldsymbol{S}$ vector $P^q = \{p_k^q,\ k = \overline{1, K}\}$, is considered the set person making decisions, (decision-maker) if everyone is set a component of this vector. Set by the decision-maker of a component p_k^q , from the point of view of the decision-maker, shows in how many time a relative estimate of $\lambda_k(X)$, k = $\overline{1, K}$ is more than other relative estimates of $\lambda_k(X)$, k = $\overline{1, K}$. The vector of p_k^q , $k = \overline{1, K}$ is the set numerical expression of a priority of q-th of criterion over other criteria of $k = \overline{1, K}$

p_k^q, $k = \overline{1, K}$, $q \in \boldsymbol{K}$, (8)

$p_k^q \geq 1$, $\forall X \in S_q \subset S$, $k = \overline{1, K}$, $q \in \boldsymbol{K}$.

VPMP in which the priority any of criteria is set, call VPMP with the set priority of criterion.

The problem of a task of a vector of priorities arises when it is necessary to determine $X^o \in S$ point by the set vector of priorities.

At operation of comparison of relative estimates with a priority of criterion of $q \in \boldsymbol{K}$, similarly, as well as in a task with equivalent criteria, we will enter the additional numerical characteristic of λ which we will call *level*.

Definition 4. The λ level is the lowest among all relative estimates with a priority of criterion of $q \in \boldsymbol{K}$, such that

$\lambda \leq p_k^q \lambda_k(X)$, $k = \overline{1, K}$, $q \in \boldsymbol{K}$, $\forall X \in S_q \subset S$;

(9)

the lower level for performance of a condition (9) is defined

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$$\lambda = \min_{k\in K} p_k^q \lambda_k(X),\ q \in \boldsymbol{K},\ \forall X \in S_q \subset S. \qquad (10)$$

Ratios (9) and (10) are interconnected and serve further as transition from operation of definition of min to restrictions and vice versa.

In the previous work [10] definition of a point of $X^o\in S$, optimum across Pareto is given. Considering this definition as initial, we will construct a number of the axioms dividing an admissible set of $\boldsymbol{S}$, first, into a subset of points of S^o, optimum across Pareto, and, secondly, on subsets of points of $S_q\subset S$, $q\in\boldsymbol{K}$, priority on q-th to criterion.

Definition 5. (**Axiom 2**) (*About a subset of points, priority by criterion*).

In VPMP (1)-(4) the subset of points of $\boldsymbol{S}_q \subset \boldsymbol{S}$ is called as area of a priority of criterion of $q\in\boldsymbol{K}$ over other criteria, if

$\forall X \in \boldsymbol{S}_q\ \forall k\in\boldsymbol{K}\ \lambda_q(X) \geq \lambda_k(X),\ q \neq k.$

This definition extends and on a set of points of $\boldsymbol{S}^o$, optimum across Pareto that is given by the following definition.

Definition 6. (**Axiom 2a**) (About a subset of points, priority by criterion, on Pareto's great number in VPMP).

In a vector problem of mathematical programming the subset of points of $\boldsymbol{S}_q^o\subset\boldsymbol{S}^o\subset\boldsymbol{S}$ is called as area of a priority of criterion of $q\in\boldsymbol{K}$ over other criteria, if

$\forall X\in\boldsymbol{S}_q^o\ \ \forall k\in\boldsymbol{K}\ \ \lambda_q(X) \geq \lambda_k(X),\ q \neq k.$

We will give some explanations.

Axiom 2 and 2a allowed to break in VPMP (1)-(4) an admissible set of points of $\boldsymbol{S}$, including a subset of points, optimum across Pareto, $\boldsymbol{S}^o \subset \boldsymbol{S}$, into subsets:

one subset of points of $\boldsymbol{S}'\in\boldsymbol{S}$ where criteria are equivalent, and a subset of points of $\boldsymbol{S}'$, being crossed with a subset of points of $\boldsymbol{S}^o$, allocates a subset of points, optimum across Pareto, at equivalent criteria of $\boldsymbol{S}^{oo} =\boldsymbol{S}'\cap\boldsymbol{S}^o$ which as it will be shown further, consists of one point of $X^o\in\boldsymbol{S}$, i.e. $X^o=\boldsymbol{S}^{oo}=\boldsymbol{S}'\cap\boldsymbol{S}^o$, $\boldsymbol{S}'\in\boldsymbol{S}$, $\boldsymbol{S}^o\in\boldsymbol{S}$;

"$\boldsymbol{K}$" of subsets of points where each criterion of $q=\overline{1,K}$ has a priority over other criteria of $k=\overline{1,K}$, $q\neq k$, thus breaks, first, sets of all admissible points of $\boldsymbol{S}$, into subsets of $\boldsymbol{S}_q\subset\boldsymbol{S}$, $q=\overline{1,K}$ and, secondly, a set of points, optimum across Pareto, $\boldsymbol{S}^o$, into subsets

$\boldsymbol{S}_q^o\subset\boldsymbol{S}^o\subset S,\ q=\overline{1,K}$,

From here the following ratios are right:

$\boldsymbol{S}'\mathrm{U}(\bigcup_{q\in K} S_q^o) = \boldsymbol{S}^o,\ \boldsymbol{S}_q^o\subset\boldsymbol{S}^o\subset\boldsymbol{S},\ q=\overline{1,K}$.

We will notice that the subset of points of $\boldsymbol{S}_q^o$ on the one hand is included in area (a subset of points) priority of criterion of $q\in\boldsymbol{K}$ over other criteria:

$$\boldsymbol{S}_q^o \subset \boldsymbol{S}_q\subset \boldsymbol{S}, \qquad (11)$$

and with another, in a subset of points, optimum across Pareto:

$$\boldsymbol{S}_q^o \subset \boldsymbol{S}^o \subset \boldsymbol{S}. \qquad (12)$$

The axiom 2 and numerical expression of a priority of criterion (7) allow to identify each admissible point of $X\in\boldsymbol{S}$ (by means of vector

$P^q(X)=\{p_k^q(X)=\lambda_q(X)/\lambda_k(X),\ k=\overline{1,K}\}$), to form and choose:

• subset of points by priority criterion of $\boldsymbol{S}_q$ which is included in a set of points of $\boldsymbol{S}$, $\forall q\in\boldsymbol{K}\ X\in\boldsymbol{S}_q\subset\boldsymbol{S}$, (such subset of points can be used in problems of a clustering, but it is beyond article);

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• subset of points by priority criterion of $\boldsymbol{S}_q^o$ which is included in a set of points of $\boldsymbol{S^o}$, optimum across Pareto, $\forall q \in K$, $X \in \boldsymbol{S}_q^o \subset \boldsymbol{S^o}$.

Thus, full identification of all points in a vector problem (1)-(4) in sequence is executed:

- Set of admissible points of $X \in \boldsymbol{S}$;
- Subset of points, optimum across Pareto, $X \in \boldsymbol{S^o} \subset \boldsymbol{S}$;
- Subset of points by priority criterion of $X \in \boldsymbol{S}_q^o \subset \boldsymbol{S^o} \subset \boldsymbol{S}$;
- Separate point of $\forall X \in \boldsymbol{S}$ $X \in \boldsymbol{S}_q^o \subset \boldsymbol{S}^o \subset \boldsymbol{S}$.

It is the most important result which will allow to output the principle of an optimality and to construct methods of a choice of any point of Pareto's great number.

Definition 7. The principle of an optimality 2 - the Solution of VPMP with the set priority of one of criteria.

VZMP with the set priority of q-th of criterion of p_k^q, $k = \overline{1,K}$ is considered solved if the point of X^o and a maximum level of λ^o among all relative estimates such that is found

$$\lambda^o = \max_{X \in S} \min_{k \in K} p_k^q \lambda_k(X), \; q \in \boldsymbol{K}. \qquad (13)$$

Using interrelation (9) and (10), we will transform a maximine problem (13) to an extreme problem of the form

$$\lambda^o = \max_{X \in S} \lambda, \qquad (14)$$

$$\lambda \le p_k^q \lambda_k(X), \; k=\overline{1,K}. \qquad (15)$$

Problem (14)-(15) we will call λ-problem with a priority of q-th of criterion.

The result of solution the λ-problem will be point $\boldsymbol{X^o}=\{X^o, \lambda^o\}$ – it is result also of the solution of VPMP (1)-(4) with the set priority of the criterion, solved on the basis of normalization of criteria and the principle of the guaranteed result. In the optimum solution of $\boldsymbol{X^o}=\{X^o, \lambda^o\}$, X^o - an optimum point, and λ^o - the maximum bottom level. The point of X^o and the λ^o level correspond to restrictions (15), which can be written as:

$$\lambda^o \le p_k^q \lambda_k(X^o), \; k \; \overline{1,K}. \qquad (16)$$

These restrictions are a basis of an assessment of correctness of results of the decision in practical vector problems of optimization.

Definition 1 and 2 "Principles of optimality" follows the opportunity to formulate the concept of the operation «opt».

Definition 8. *Mathematical operation "opt".*

In VPMP (1)-(4) which part criteria of "max" and "min" are, the mathematical operation "opt" consists in definition of a point of X^o and the maximum λ^o bottom level to which all criteria measured in relative units are lifted:

$$\lambda^o \le \lambda_k(X^o) = \frac{f_k(X) - f_k^o}{f_k^* - f_k^o}, \; k=\overline{1,K}, \qquad (17)$$

i.e. all criteria of $\lambda_k(X^o)$, $k=\overline{1,K}$ are equal or more maximum level of λ^o, (therefore λ^o also is called as the guaranteed result).

Theorem 1. *The theorem of the most inconsistent criteria in VPMP with the set priority*.

If in a convex vector problem of mathematical programming of maximizing (1)-(4) the priority of q-th of criterion of p_k^q, $k = \overline{1,K}$, $\forall q \in \boldsymbol{K}$ over other criteria is set, in a point of an optimum of $X^o \in \boldsymbol{S}$ received on the basis of normalization of criteria and the principle of guaranteed result, always there will be two criteria with the indexes $r \in \boldsymbol{K}$, $t \in \boldsymbol{K}$, for which strict equality is carried out:

$\lambda^o = p_k^r \lambda_r(X^o) = p_k^t \lambda_t(X^o)$, $r, t, \in \boldsymbol{K}$,

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(18)

and other criteria are defined by inequalities:

$\lambda^o \le p_k^q(X^o)$, $k = \overline{1,K}$, $\forall q \in \boldsymbol{K}$, $q \neq r \neq t$

. (19)

Criteria with the indexes $r \in \boldsymbol{K}$, $t \in \boldsymbol{K}$ for which equality (18) is carried out are called the most inconsistent.

Proof. Similar to the theorem 2 [8].

We will notice that in (18) and (19) indexes of criteria of r, $t \in \boldsymbol{K}$ can coincide with the $q \in \boldsymbol{K}$ index.

Consequence of the theorem 1. *About equality of an optimum level and relative estimates in VPMP with two criteria with a priority of one of them*.

In a convex vector problem of mathematical programming with two equivalent criteria, solved on the basis of normalization of criteria and the principle of the guaranteed result, in an optimum point of X^o equality is always carried out:

at a priority of the first criterion over the second:

$\lambda^o = \lambda_1(X^o) = p_2^1(X^o)\lambda_2(X^o)$, $X^o \in \boldsymbol{S}$,

(20)

where $p_2^1(X^o) = \lambda_1(X^o)/\lambda_2(X^o)$

at a priority of the second criterion over the first:

$\lambda^o = p_1^2(X^o)\lambda_1(X^o) = \lambda_2(X^o)$, $X^o \in \boldsymbol{S}$, где $p_1^2(X^o) = \lambda_1(X^o)/\lambda_2(X^o)$

Algorithm of the decision in problems of vector optimization with a criterion priority

The axiomatic of vector optimization presented to [10] and in section 3.1, and also algorithm of the solution of vector problem at equivalent criteria, allow to conduct research of all structure of a great number of Pareto and to choose a necessary point, i.e. to make the optimum decision on the model presented by VPMP.

The vector problem (1)-(4) is considered. It is supposed that in this problem there is a criterion $q \in \boldsymbol{K}$ priority. With a priority of criterion we understand receiving an optimum point from criterion priority area according to an axiom 2 and the set vector of priorities as the solution of VPMP.

Usually at the solution of vector problem with priorities use knowledge of the person to whom the physical sense of a problem is clear and the priority (preference) of this or that criterion over other criteria is clear, such person call the person making decisions - the decision-maker.

In the offered technique for the person making decisions leave a choice of priorities of criterion, and procedure of obtaining the optimum decision at equivalent criteria, calculation of limits of change the component of a vector of priorities and the solution of task □ are carried out on the Computer, i.e. human-machine procedure of decision-making for the models presented by a vector problem of mathematical programming is carried out.

The choice of a point of $X^o \in \boldsymbol{S}$ is carried out on two algorithms which differ from each other by specifying of basic data:

- the vector of priorities of p_k^q, $k=\overline{1,K}$, $q \in \boldsymbol{K}$ is set, and is the corresponding point of an optimum of X^o at which the size $f_q(X^o)$ comes nearer to set f_q^0, see definitions 1-3;
- the size of criterion function of f_q^0 is set, for it the vector of priorities of p_k^q, $k=\overline{1,K}$, $q \in \boldsymbol{K}$ is defined, and is calculated $X^o \in \boldsymbol{S}$ point.

Algorithm of the solution of VPMP with a criterion priority.

Step 1. VPMP with equivalent criteria, see [10] is solved.

As a result of the decision we will receive:

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optimum points by each criterion separately X_k^*, $k=\overline{1,K}$ and sizes of criterion functions in these points of $f_k^*=f_k(X_k^*)$, $k=\overline{1,K}$ which represent boundary of a set of points, optimum across Pareto;

anti-optimum points by each criterion of $X_k^0=\{x_j, j=\overline{1,N}\}$ and the worst unchangeable part of criterion of $f_k^0=f_k(X_k^0)$, $k=\overline{1,K}$;

$\boldsymbol{X^o}=\{\lambda^o, X^o\}$ - an optimum point, as result of the solution of VPMP at equivalent criteria, i.e. result of the solution of a maximine problem and λ-problem constructed on its basis;

λ^o - the maximum relative assessment which is the maximum lower level for all relative estimates of $\lambda_k(X^o)$, or the guaranteed result in relative units, λ^o guarantees that all relative estimates of $\lambda_k(X^o)$ more or are equal in X^o point to λ^o:

$$\lambda^o \le \lambda_k(X^o),\ k=\overline{1,K},\ X^o \in S. \tag{21}$$

The person making the decision, carries out the analysis of results of the solution of VPMP at equivalent criteria (The block 5, fig. 1 [10]). If the received results satisfy the decision-maker, the end (transition to the block 8, fig. 1 [10]), differently the subsequent calculations.

We will in addition calculate:

• in each point of X_k^*, $k=\overline{1,K}$ we will determine sizes of all criteria of $q=\overline{1,K}$: $\{f_q(X_k^*), q=\overline{1,K}\}$, $k=\overline{1,K}$, and relative estimates

$\lambda(X^*)=\{\lambda_q(X_k^*), q=\overline{1,K}, k=\overline{1,K}\}$, $\lambda_k(X)=\dfrac{f_k(X)-f_k^o}{f_k^*-f_k^o}$, $\forall k\in \boldsymbol{K}$:

$$F(X^*)=\begin{vmatrix} f_1(X_1^*),\dots,f_k(X_1^*),\\ \dots \\ f_1(X_k^*),\dots,f_k(X_k^*)\end{vmatrix},\quad \lambda(X^*)=\begin{vmatrix} \lambda_1(X_1^*),\dots,\lambda_k(X_1^*),\\ \dots \\ \lambda_1(X_k^*),\dots,\lambda_k(X_k^*)\end{vmatrix}. \tag{22}$$

Matrixes of criteria of $F(X^*)$ and relative estimates of $\lambda(X^*)$ show sizes of each criterion of $k=\overline{1,K}$ upon transition from one optimum point of X_k^*, $k\in\boldsymbol{K}$ to another to X_q^*, $q\in\boldsymbol{K}$, i.e. on border of a great number of Pareto.

• in an optimum point at equivalent criteria of X^o we will calculate sizes of criteria and relative estimates:

$$f_k(X^o),\ k=\overline{1,K};\ \ \lambda_k(X^o),\ k=\overline{1,K}, \tag{23}$$

which satisfy to an inequality (21). In other points of $X\in\boldsymbol{S^o}$ smaller of criteria in relative units of $\lambda=\min_{k\in K}\lambda_k(X)$ is always less than λ^o.

Are remembered given λ-problem (19)-(20) of [10].

$$\lambda^o = \max \lambda, \tag{24}$$

$$\lambda - \frac{f_k(X)-f_k^o}{f_k^*-f_k^o} \le 0,\ k=\overline{1,K}, \tag{25}$$

$$G(X)\le B,\ X\ge 0, \tag{26}$$

where the vector of unknown of X has dimension of $N+1$: $X=\{\lambda, x_1, \dots, x_N\}$.

This information also is a basis for further studying of structure of a great number of Pareto.

Step 2. Choice of priority criterion of $q\in\boldsymbol{K}$.

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From the theory (see the theorem 1, [10]) it is known that in an optimum point of X^o always there are two most inconsistent criteria, $q\in\boldsymbol{K}$ and $v\in\boldsymbol{K}$ for which in relative units exact equality is carried out:

$$\lambda^o = \lambda_q(X^o) = \lambda_p(X^o),\ q,\ v \in \boldsymbol{K},\ X\in\boldsymbol{S}, \tag{27}$$

and for the others it is carried out inequalities: $\lambda^o \leq \lambda_k(X^o)\ \ \forall k \in \boldsymbol{K},\ q \neq v \neq k$.

As a rule, the criterion which the decision-maker would like to improve gets out of this couple, such criterion is called as "priority criterion", we will designate it $q\in\boldsymbol{K}$.

Step 3. Numerical limits of change of size of a priority of criterion of $q\in\boldsymbol{K}$ are defined.

For priority criterion of $q\in\boldsymbol{K}$ from a matrix (22) we will define numerical limits of change of size of criterion:

• in physical units of $f_q(X^o)\leq f_q(X)\leq f_q(X^*_q),\ k\in\boldsymbol{K}$,

$$\tag{28}$$

where $f_q(X^*_q)$ undertakes from a matrix (22) $F(X^*)$, all criteria showing sizes measured in physical units;

$f_q(X^o)$ from (23);

• in relative units of $\lambda_q(X^o)\leq \lambda_q(X)\leq \lambda_q(X^*_q),\ k\in\boldsymbol{K}$,

$$\tag{29}$$

where $\lambda_q(X^*_q)$ undertakes from a matrix (22) $\lambda(X^*)$, all criteria showing sizes measured in relative units (we will notice that $\lambda_q(X^*_q)=1$);

$\lambda_q(X^o)$ from (23).

As a rule, results (28)-(29) are given for the display for the analysis.

Step 4. Limits of change of a vector of a priority of criterion of P^q, $q\in\boldsymbol{K}$ are defined.

The size of a priority of criterion of $q\in\boldsymbol{K}$ in relation to other criteria is determined in points of X^o and X^*_q by formulas:

$$p^q_k(X^o) = \frac{\lambda_q(X^o)}{\lambda_k(X^o)},\ p^q_k(X^*_q) = \frac{\lambda_q(X^*_q)}{\lambda_k(X^*_q)},\ k = \overline{1,K}$$

also limits of change of the set vector of priorities are set

$$p^q_k(X^o) \leq p^q_k \leq p^q_k(X^*_q),\ k=\overline{1,K},\ \forall q\in K. \tag{30}$$

Results of calculations are given for the press (on the display):

$$f^*_1,\qquad f^*_2,\ \ldots,\qquad f^*_k, \tag{31}$$

$f_1(X^o),\ \ f_2(X^o),\ \ldots,\ \ f_k(X^o)$,

$f_1(X^*_q),\ \ f_2(X^*_q),\ \ldots,\ \ f_k(X^*_q)$,

$\lambda_1(X^*_q),\ \lambda_2(X^*_q),\ \ldots,\ \lambda_k(X^*_q)$

and limits of change of the set vector of priorities (30).

These results also form a basis for decision-making.

Step 5. Choice of the priority characteristic of the TS and its sizes (Decision-making).

The person making the decision, carries out the analysis of results of calculations (31) and from an inequality (28) chooses the numerical size f_q of criterion of $q\in\boldsymbol{K}$:

$$f_q(X^o) \leq f_q \leq f_q(X^*_q),\ k\in\boldsymbol{K}. \tag{32}$$

For the chosen size of criterion of f_q it is necessary to define a vector of unknown X^{oo}, for this purpose we carry out the subsequent calculations.

Step 6. Calculation of a relative assessment and vector of priorities.

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For the chosen size of priority criterion of f_q the relative assessment is calculated:

$$\lambda_q = \frac{f_q - f_q^o}{f_q^* - f_q^o}, \qquad (33)$$

which upon transition from X^o point to X_q^* according to (29) lies in limits:

$\lambda_q(X^o) \leq \lambda_q \leq \lambda_q(X_q^*) = 1.$

Assuming linear nature of change of criterion of $f_q(X)$ in (26) and according to a relative assessment of $\lambda_q(X)$ in (29), using standard methods of linear approximation, we will calculate proportionality coefficient between $\lambda_q(X^o)$, λ_q, which we will call ρ:

$$\rho = \frac{\lambda_q - \lambda_q(X^o)}{\lambda_q(X_q^*) - \lambda_q(X^o)}, \; q \in \boldsymbol{K}. \qquad (34)$$

Assuming linear nature of change of a vector of P^q in (30) and using the received coefficient of proportionality of ρ, we will calculate all components of a vector of priorities of $P^q = \{p_k^q, \; k = \overline{1,K}\}$,

$$p_k^q = p_k^q(X^o) + \rho(p_k^q(X_q^*) - p_k^q(X^o)), \; k = \overline{1,K}, \; q \in \boldsymbol{K}. \qquad (35)$$

as a result received "a settlement vector of priorities" which identifies future point of an optimum of X_t^o with an error of linear approximation of real change of $q \in \boldsymbol{K}$ of criterion.

Step 7. Construction and the solution of λ-problem with a criterion priority.

The set vector of priorities (35) is entered into data of λ-problem (24)-(26) as a result received λ-problem with a criterion priority:

$$\lambda^o = \max \lambda, \qquad (36)$$

$$\lambda - p_k^q \frac{f_k(X) - f_k^o}{f_k^* - f_k^o} \leq 0, \; k = \overline{1,K}, \qquad (37)$$

$$G(X) \leq B, \; X \geq 0, \qquad (38)$$

As a result of the solution of task λ-problem (36)-(38) are received:

$\boldsymbol{X}_t^o = \{\lambda_t^o, X_t^o\}$ - an optimum point;

λ_t^o - maximum relative assessment;

$f_k(X_t^o)$, $k = \overline{1,K}$ - sizes of all criteria in an optimum point;

$\lambda_k(X_t^o)$, $k = \overline{1,K}$ - sizes of all relative estimates for which inequalities are right: $\lambda_t^o \leq p_k^q \lambda_k(X_t^o)$, $k = \overline{1,K}$, $\forall q \in \boldsymbol{K}$;

$p_k^q(X_t^o)$, $k = \overline{1,K}$ - a criterion q-th priority vector over other criteria of $k = \overline{1,K}$.

Step 8. Analysis of results.

The calculated size $f_q(X_t^o)$, $q \in \boldsymbol{K}$ is usually not equal to the set f_q. The error of a choice of f_q is defined by an error of linear approximation of the valid change of relative estimates of $\lambda_k(X)$, $k = \overline{1,K}$ and a vector of priorities of $p_k^q(X)$, $k = \overline{1,K}$, $q \in \boldsymbol{K}$ executed on steps 5, 6.

If error $\Delta f_q = |f_q(X_t^o) - f_q|$ is less than set Δf, we pass to a step 10 if $\Delta f_q \geq \Delta f$, is carried out the following step.

Шаг 9. В точке X^o определяется вектор приоритетов критерия $q \in \boldsymbol{K}$.

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$p_k^q(X^o) = \lambda_q(X^o)/\lambda_k(X^o)$, $k=\overline{1,K}$, $q\in \boldsymbol{K}$.

Строится новый вектор приоритетов:

$p_{kw}^q = (p_k^q + p_k^q(X^o))/2$, $k=\overline{1,K}$, $q\in \boldsymbol{K}$.

Переход к шагу 7. При этом λ-задача строится с новым вектором приоритетов p_{kw}^q, $k=\overline{1,K}$, $q\in \boldsymbol{K}$ вместо p_k^q, $k=\overline{1,K}$, $q\in \boldsymbol{K}$.

Step 9. In a point of X^o the vector of priorities of criterion of $q\in \boldsymbol{K}$ is defined.

The new vector of priorities is under construction:

Transition to a step 7. Thus λ-problem is under construction with a new vector of priorities of p_{kw}^q, $k=\overline{1,K}$, $q\in \boldsymbol{K}$ instead of p_k^q, $k=\overline{1,K}$, $q\in \boldsymbol{K}$.

Step 10. Whether VPMP for other size of priority criterion will be solved? If "yes", transition to a step 3 if "no", the following step is carried out.

Step 11. Whether VPMP with other priority criterion will be solved? If "yes", transition to a step 2, differently the following step.

Step 12. End.

As a result of the solution of VPMP (1)-(4) and the corresponding λ-problem (36)-(38) with a priority of q-th of criterion received:

- $\boldsymbol{X^{oo}}=\{X^{oo}, \lambda^{oo}\}$ - an optimum point which represents design data of the TS and the maximum relative assessment of λ^o such that $\lambda^o \le p_k^q \lambda_k(X_t^o)$, $k=\overline{1,K}$, $\forall q\in \boldsymbol{K}$ where a vector of priorities of p_k^q, $k=\overline{1,K}$ corresponds to the set concepts decision-makers about a priority of q-th of criterion over the others;
- $f_k(X^{oo})$, $k=\overline{1,K}$ - sizes of criteria (TS characteristics);
- $\lambda_k(X^{oo})$, $k=\overline{1,K}$ - sizes of relative estimates;
- λ^{oo} – the maximum lower level among all relative estimates, measured in relative units: $\lambda^{oo}=\min(p_k^q \lambda_k(\boldsymbol{X^{oo}}), k=\overline{1,K})$.

λ^{oo} – also call the guaranteed result in relative units, i.e. $\lambda_k(\boldsymbol{X^{oo}})$ and according to the characteristic of the $f_k(\boldsymbol{X^{oo}})$ TS it is impossible to improve, without worsening thus other characteristics.

It is supposed that in this algorithm each step is carried out on the Computer, and sizes a component of a vector of priorities are set by the decision-maker, thus on a step 4 comparison of the received point of $\boldsymbol{X^{oo}}$ and maximum by X_q^* in size q-th of criterion function is made. (It is clear that is better than points of $\boldsymbol{X^{oo}}$ and by X_q^* it can't be received, i.e. they are optimum across Pareto).

The optimal decision on the model of the technical system with the priority criteria

The model of technical system for which are known is considered:

• functional dependence of each characteristic (criterion) of $k\in \boldsymbol{K}$ (1)-(2) on a vector of design data (variables) of $X=\{x_1, x_2\}$, the set of criteria $\boldsymbol{K}$=4 is divided into two under sets - two criteria of max and two min;

• restrictions (3)-(4) which are functionally dependent on the same TS parameters.

The model of technical system is presented by a vector problem of the mathematical (nonlinear) programming which numerical values correspond [10]:

$$\boldsymbol{opt\ F(X)}=\{max\ F_1(X)=\{max\ f_1(X) \equiv 639+0.031x_1+0.0421x_1^2+0.039x_2^2, \quad (39)$$

$$max\ f_3(X) \equiv 402 - 0.04x_1 + 0.04x_1^2 + 0.08x_2^2\}, \quad (40)$$

$$min\ F_2(X)=\{min\ f_2(X) \equiv -506 + 0.71x_1+528x_2 - 1.9x_2^2, \quad (41)$$

$$min\ f_4(X) \equiv -803+0.203x_1+0.135x_1^2 - 0.09x_1x_2\}\}, \quad (42)$$

at restrictions

$$10000 \le f_2(X) \equiv -506+0.71x_1+528x_2-0.19x_2^2 \le 21000, \quad (43)$$

$$10\le x_1\le 80,\ 15\le x_2\le 70. \quad (44)$$

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The methodology of modeling and adoption of the optimum decision in the annex to TS (39)-(44) model at the set priority of criterion, is based on axiomatic, with use of normalization of criteria and the principle a maximine, according to a technique section 3.2, and presented in the form of sequence of steps.

Step 1. VPMP with equivalent criteria, see is solved [10].

As a result of the decision we will receive:

• optimum point by each criterion separately X_k^*, $k=\overline{1,4}$ and sizes of criterion functions in these points of $f_k^*=f_k(X_k^*)$, $k=\overline{1,4}$:

$f_1(X)\rightarrow max$ $X_1^*=\{x_1=80.0, x_2=48.5479\}$, $f_1^*=f_1(X_1^*)=-1003.4$;

$f_2(X)\rightarrow min$ $X_2^*=\{x_1=12.7076, x_2=21.55\}$, $f_2^*=f_2(X_2^*)=1000$;

$f_3(X)\rightarrow max$ $X_3^*=\{x_1=80.0, x_2=49.4079\}$, $f_3^*=f_3(X_3^*)=-850.0912$;

$f_4(X)\rightarrow min$ $X_4^*=\{x_1=15.7512, x_2=49.5421\}$, $f_4^*=f_4(X_4^*)=-836.7402$;

• anti-optimum points by each criterion of $X_k^0=\{x_j, j=\overline{1,N}\}$ and the worst unchangeable part of criterion of $f_k^0=f_k(X_k^0)$, $k=\overline{1,4}$:

$f_1(X)\rightarrow min$ $X_1^o=\{x_1=10.0, x_2=21.5564\}$, $f_1^o=f_1(X_1^o)=661.64$;

$f_2(X)\rightarrow max$ $X_2^o=\{x_1=30.0543, x_2=49.5122\}$, $f_2^o=f_2(X_2^o)=-21000.0$;

$f_3(X)\rightarrow min$ $X_3^o=\{x_1=10.0, x_2=21.5564\}$, $f_3^o=f_3(X_3^o)=442.7743$;

$f_4(X)\rightarrow max$ $X_4^o=\{x_1=80.0, x_2=45.4499\}$, $f_4^*=f_4(X_4^*)=250.1992$;

• as a result of the solution of VPMP (29)-(34) at equivalent criteria and to λ-problem corresponding to it (37)-(42) we will receive:

$\boldsymbol{X^o}=\{X^o, \lambda^o\}=\{x_1=57.7806, x_2=35.4081, \lambda^o=-0.4683\}$ - an optimum point which represents design parameters of the TS and the maximum relative assessment of $\lambda^o=0.4683$;

$f_k(X^o)$, $k=\overline{1,K}$ - sizes of criteria (TS characteristics)

$f_k(X^o)=\{-830\ 15848\ -634\ -525\}$; (45)

$\lambda_k(X^o)$, $k=\overline{1,K}$ - sizes of relative estimates

$\lambda_k(X^o)=\{0.4933\ 0.4683\ 0.4683\ 0.4683\}$; (46)

$\lambda^o=0.4683$ is the maximum bottom level among all relative estimates, measured in relative units: $\lambda^o=\min(\lambda_1(X^o), \lambda_2(X^o), \lambda_3(X^o), \lambda_4(X^o))=0.4683$, λ^o – also call the guaranteed result in relative units, i.e. $\lambda_k(X^o)$ and according to the characteristic of the $f_k(X^o)$ TS it is impossible to improve, without worsening thus other characteristics.

We will in addition calculate:

• in each point of X, = we will determine by k sizes of all criteria of $q=\overline{1,K}$:

$\{f_q(X_k^*), q=\overline{1,K}\}$, $k=\overline{1,K}$, $K=4$ and relative estimates $\{\lambda_q(X_k^*), q=\overline{1,K}\}$, $k=\overline{1,K}$.

$$F(X^*)=\begin{vmatrix} 1003 & 20759 & 845 & -274 \\ 664 & 9999 & 445 & -803 \\ 1006 & 21000 & 850 & -279 \\ 746 & 21000 & 608 & -837 \end{vmatrix}, \quad (47)$$

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$$\lambda(X^*)=\begin{vmatrix} 1.0000 & 0.0219 & 0.9864 & 0.0399 \\ 0.0078 & 1.0000 & 0.0057 & 0.9433 \\ 1.0079 & 0 & 1.0000 & 0.0486 \\ 0.2458 & 0 & 0.4048 & 1.0000 \end{vmatrix}. \quad (48)$$

$d1$ = 341.7786, $d2$ = -11001, $d3$ = 407.32, $d4$ = -586.54

From matrix $\lambda(X^*)$ follows that in private optimum points relative estimates reach the greatest sizes and are equal to unit.

Matrixes of criteria of $F(X^*)$ and relative estimates of $\lambda(X^*)$ show sizes of each criterion of $k=\overline{1,K}$ upon transition from one optimum point of X^*_k, $k\in\boldsymbol{K}$ to another to X^*_q, $q\in\boldsymbol{K}$, i.e. on border of a great number of Pareto

• in an optimum point at equivalent criteria of X^o we will calculate sizes of criteria and relative estimates:

$$f_k(X^o),\ k=\overline{1,K};\ \lambda_k(X^o),\ k=\overline{1,K}, \quad (49)$$

which satisfy to an inequality (21). In other points of $X\in\boldsymbol{S}^o$ smaller of criteria in relative units of $\lambda = \min_{k\in K} \lambda_k(X)$ is always less than λ^o.

Data of λ-problem (19)-(20) of are remembered [10].

$$\lambda^o = \max \lambda, \quad (50)$$

$$\lambda - \frac{f_k(X) - f_k^o}{f_k^* - f_k^o} \le 0,\ k = \overline{1,K},\ K=4, \quad (51)$$

$$G(X) \le B,\ X \ge 0, \quad (52)$$

where the vector of unknown of X has dimension of N+1: $X=\{\lambda, x_1, \dots, x_N\}$.

For an assessment of a set of points, optimum across Pareto we will calculate auxiliary points: X^o_{12}, X^o_{13}, X^o_{34}, X^o_{42} which are received by a solution of λ-problem

Criteria 1, 2: X^o_{12}={80.0 25.96}, $\lambda_2(X^o_{12})=\lambda_1(X^o_{12})$=-0.8063,

Criteria 1, 3: X^o_{13}={80.0 48.97}, $\lambda_1(X^o_{13})=\lambda_3(X^o_{13})$=-0.9916,

Criteria 3, 4: X^o_{34}={55.16 46.42}, $\lambda_3(X^o_{34})=\lambda_4(X^o_{34})$=-0.616,

Criteria 4, 2: X^o_{42}={10.0 22.76}, $\lambda_4(X^o_{42})=\lambda_2(X^o_{42})$= -0.9513

In fig. 1 the admissible set of points of $\boldsymbol{S}$ formed by restrictions (44), points of private optimum is shown: X^*_1, X^*_2, X^*_3, X^*_4; auxiliary points: X^o_{12}, X^o_{13}, X^o_{34}, X^o_{42}. All these points are united in a contour which represents a set of points, optimum across Pareto, $\boldsymbol{S}^o \subset \boldsymbol{S}$.

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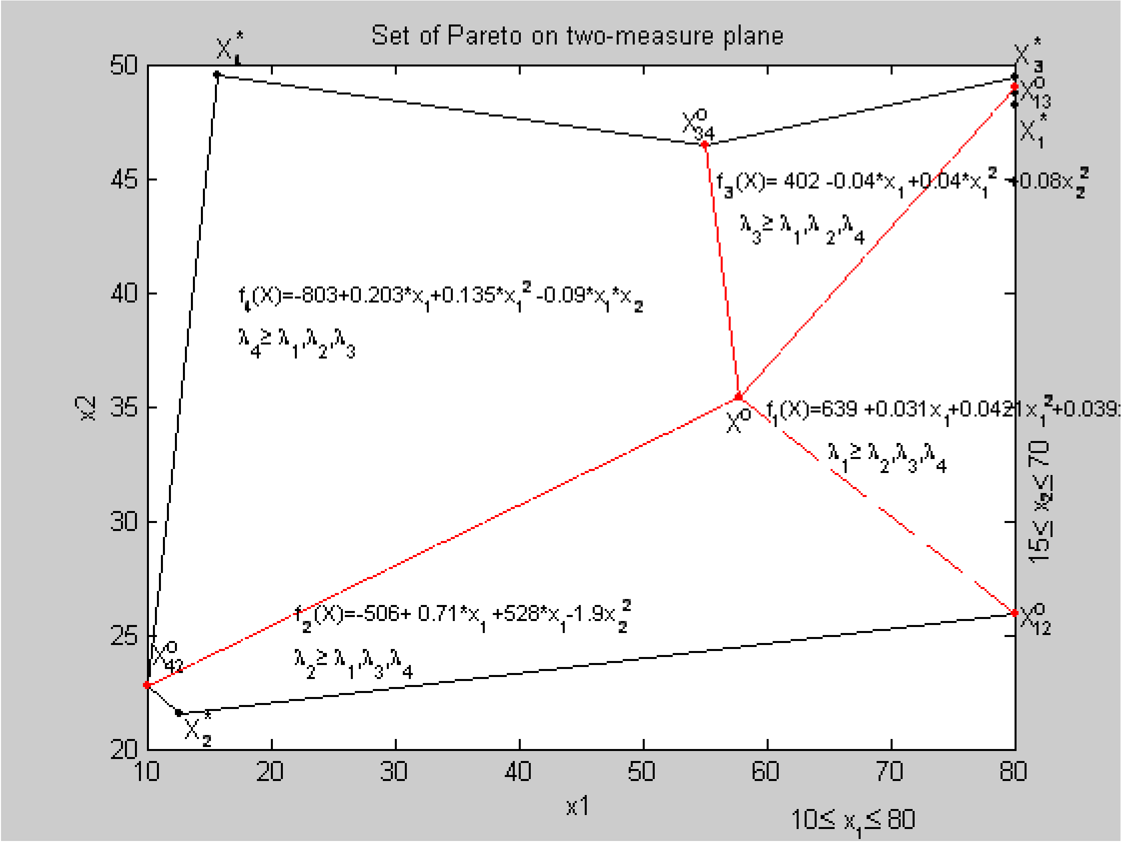

Fig. 1. Admissible set of points S of problem (39)-(44) and a set of points to be Pareto optimal $S^o \subset S$ in two-dimensional system of coordinates of x_1 and x_2.

We will carry out their analysis. For this purpose we will connect auxiliary points: $X_{12}^o, X_{13}^o, X_{34}^o, X_{42}^o$ with a point $\boldsymbol{X^o}$ which conditionally represents the center of a great number of Pareto. As a result received four subsets of points of $X \in \boldsymbol{S}_q^o \subset \boldsymbol{S}^o \subset \boldsymbol{S}$, $q=\overline{1,4}$. The Subset of $\boldsymbol{S}_1^o \subset \boldsymbol{S}^o \subset \boldsymbol{S}$ is characterized by that the relative assessment of $\lambda_1 \geq \lambda_2, \lambda_3$, λ_4, i.e. in the field of $\boldsymbol{S}_1^o$ first criterion has a priority over the others. Similar to S $\boldsymbol{S}_2^o$, $\boldsymbol{S}_3^o$, $\boldsymbol{S}_4^o$ - subsets of points where the second, third and fourth criterion has a priority over the others respectively. A set of points, optimum according to Pareto $\boldsymbol{S}^o = \boldsymbol{S}_1^o \cup \boldsymbol{S}_2^o \cup \boldsymbol{S}_3^o \cup \boldsymbol{S}_4^o$. The coordinates of all received points and relative estimates presented in fig. 1, are shown in three measured space $\{x_1, x_2, \lambda\}$ from a point of X_3^* in fig. 2, where the third axis of λ - a relative assessment. Restrictions of a set of points, optimum across Pareto, in fig. 2 it is lowered to -0.5 (that restrictions was visible).

This information is also a basis for further studying of structure of a great number of Pareto.

If results of the solution of VPMP at equivalent criteria (The block 5, fig. 1 [10]) don't satisfy the person making the decision, the choice of the optimum decision is carried out from any subset of points of $\boldsymbol{S}_1^o$, $\boldsymbol{S}_2^o$, $\boldsymbol{S}_3^o$, $\boldsymbol{S}_4^o$.

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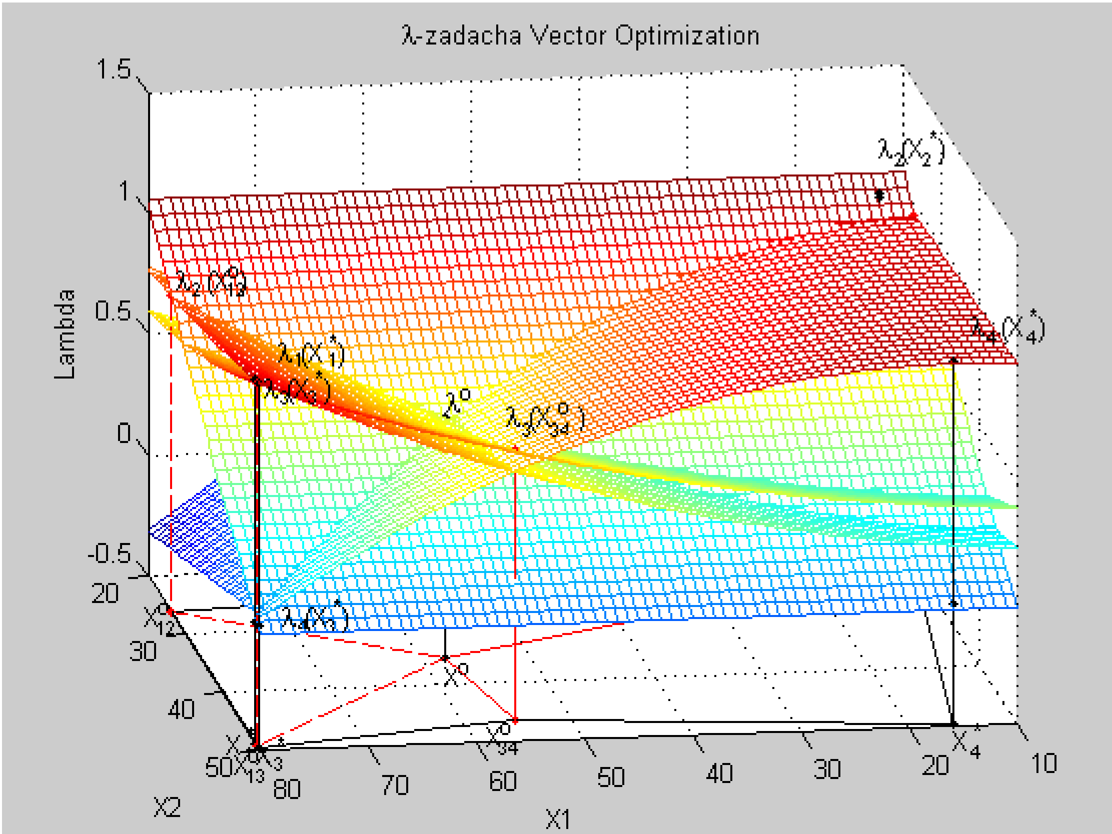

Fig. 2. λ-problem and points of an optimum of VPMP (39)-(44) in three-dimensional system of coordinates of x_1, x_2 and λ.

Step 2. Choice of priority criterion of $q \in \boldsymbol{K}$.

From the theory (see the theorem 1, [10]) it is known that in an optimum point of X^o always there are two most inconsistent criteria, $q \in \boldsymbol{K}$ and $v \in \boldsymbol{K}$ for which in relative units exact equality is carried out:

$\lambda^o = \lambda_q(X^o) = \lambda_p(X^o)$, $q, v \in \boldsymbol{K}$, $X \in \boldsymbol{S}$,

and for the others it is carried out inequalities: $\lambda^o \leq \lambda_k(X^o)$ $\forall k \in \boldsymbol{K}$, $q \neq v \neq k$.

In model of the TS (39)-(44) and the corresponding λ-problem (37)-(42) such criteria from (46) are the second, the third and the fourth:

$\lambda^o = \lambda_2(X^o) = \lambda_3(X^o) = \lambda_4(X^o) = 0.4683$, (53)

As a rule, the criterion which the decision-maker would like to improve gets out of this couple, such criterion is called as "priority criterion", we will designate it $q=3 \in \boldsymbol{K}$. This criterion is investigated in interaction with the fourth criterion of $k=4 \in \boldsymbol{K}$. We will allocate these two criteria from all set of the criteria $\boldsymbol{K}$=4 shown in fig. 2. We will present criteria of q=3 k=4 in separate drawing of fig. 3 in order that was the picture of construction and a choice of size of priority criterion is visible.

The message is issued for the display:

q=input("Enter priority criterion (number)' q=) - Entered: q=3.

Step 3. Numerical limits of change of size of a priority of criterion of $q=3 \in \boldsymbol{K}$ are defined.

For priority criterion of $q \in \boldsymbol{K}$ from a matrix (47) numerical limits of change of size of criterion are defined and are given the message for the screen:

limits of priority criterion in physical units:

$f_q(X^o) = 633.53 \leq f_q(X) \leq 850.09 = f_q(X^*_q)$, $q=3 \in \boldsymbol{K}$; (54)

in relative units $\lambda_q(X^o) = 0.4683 \leq \lambda_q(X) \leq 1 = \lambda_q(X^*_q)$, $q=3 \in \boldsymbol{K}$.

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These data it is analyzed. On the message: "Enter the size of priority criterion of *fq* =" - we enter, for example, f_q =750.

Шаг 4. *Определяются пределы изменения вектора приоритета критерия* P^q, $q\in \mathbf{K}$.

Для выбранной величины приоритетного критерия f_q=750 вычисляется относительная оценка:

$$\lambda_q = \frac{f_q - f_q^o}{f_q^* - f_q^o} = \frac{750 - 442.77}{850.09 - 442.77} = 0.7543, \quad (55)$$

Step 4. Limits of change of a vector of a priority of criterion of P^q, $q\in \mathbf{K}$ are defined.

For the chosen size of priority criterion of f_q =750 the relative assessment is calculated:

$$\lambda_q = \frac{f_q - f_q^o}{f_q^* - f_q^o} = \frac{750 - 442.77}{850.09 - 442.77} = 0.7543, \quad (55)$$

which upon transition from X^o point to X_3^* lies in limits:

$0.4683 = \lambda_3(X^o) \le \lambda_3 = 0.7543 \le \lambda_3(X_3^*) = 1$, $q\in \mathbf{K}$.

Step 5. Assuming linear nature of change of criterion of $f_q(X)$ in (54) and according to a relative assessment of λ_q in (55), using standard methods of linear approximation, we will calculate proportionality coefficient between $\lambda_q(X^o)$, λ_q which we will call ρ:

$$\rho = \frac{\lambda_q - \lambda_q(X^o)}{\lambda_q(X_q^*) - \lambda_q(X^o)} = \frac{0.7543 - 0.4683}{1 - 0.4683} = 0.5378, \; q=3\in \mathbf{K}. \quad (56)$$

Step 6. Assuming linear nature of change of a vector of P^q in (35)

$p_k^q = p_k^q(X^o) + \rho(p_k^q(X_q^*) - p_k^q(X^o))$, $k=\overline{1,K}$, $q\in \mathbf{K}$,

and using the received coefficient of proportionality of ρ, we will calculate all components of a vector of priorities of $P^q = \{p_k^q, k=\overline{1,K}\}$,

P^q=[0.9724 1000. 1.0 11.5315]. (57)

XPq=[Xo(1)+(x3max(1)-Xo(1))*Kp Xo(2)+(x3max(2)-Xo(2))*Kp]

The initial (starting) point can be similarly calculated, the operator in Matlab will assume an air:

XPq=[Xo(1)+(x3max(1)-Xo(1))*Kp Xo(2)+(x3max(2)-Xo(2))*Kp]

where Kp=ρ. The starting point is shown in fig. 3 on direct $X^oX_3^*$ which lies at the level of z =-0.5.

Step 7. *Construction and the solution of λ-problem with a criterion priority*.

The vector of priorities (57) is entered into data of λ-problem (24)-(26) as a result received λ-problem with a criterion priority:

$$\lambda^o = \max \lambda, \quad (58)$$

$$\lambda - 0.9724\frac{639 + 0.031x_1 + 0.0421x_1^2 + 0.039x_2^2 - f_1^o}{f_1^* - f_1^o} \le 0, \quad (59)$$

$$\lambda - 1000\frac{402 - 0.04x_1 + 0.04x_1^2 + 0.08x_2^2 - f_2^o}{f_2^* - f_2^o} \le 0, \quad (60)$$

$$\lambda - 1.0\frac{-506 + 0.71x_1 + 528x_2 - 0.19x_2^2 - f_3^o}{f_3^* - f_3^o} \le 0, \quad (61)$$

$$\lambda - 11.53\frac{-803.2 + 0.203x_1 + 0.135x_1^2 - 0.09x_1x_2 - f_4^o}{f_4^* - f_4^o} \le 0, \quad (62)$$

при ограничениях at restrictions

$10000 \le f_2(X) \le 21000$; $0 \le \lambda \le 1$, $10\le x_1\le 80$, $15\le x_2\le 70$, (63)

where the vector of unknown has dimension of N+1: $\mathbf{X}=\{x_1, x_2, \lambda\}$.

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The appeal to function of decisive λ-problem (58)-(63) it is representable as:

```
[Xoo,Loo] =
fmincon('TehnSist_L',X0,Ao,bo,Aeq,beq,lbo,ubo,'TehnSist_LConst1',options)
```

where result of the solution of Xoo= $\boldsymbol{X^{oo}}$, Loo =λ^{oo}.

As a result of the solution of VPMP (29)-(34) with a priority of the third criterion and to λ-problem corresponding to it (58)-(63) we will receive:

- $\boldsymbol{X^{oo}}=\{X^{oo}, \lambda^{oo}\}=\{x_1=76.5478, x_2=42.9348, \lambda^o= 0.8299\}$ - an optimum point which represents design data of the TS and the maximum relative assessment λ^o =-0.8299;
- $f_k(X^{oo})$, $k=\overline{1,K}$ - sizes of criteria (TS characteristics) $f_k(X^{oo})$={-960 18715 -781 -292};
- $\lambda_k(X^{oo})$, $k=\overline{1,K}$ - sizes of relative estimates of $\lambda_k(X^{oo})$={ 0.8728 0.2077 0.8299 0.0720};
- λ^{oo}=0.8299 is the maximum bottom level among all relative estimates, measured in relative units:

$$\lambda^{oo}=\min (p_1^3\lambda_1(X^{oo}), p_2^3\lambda_2(X^{oo}), p_3^3\lambda_3(X^{oo}), p_4^3\lambda_4(X^{oo})) =$$
$$\min (0.8487\ \ 207.6859\ \ \ 0.8299\ \ \ 0.8299)=0.\ 8299,$$

λ^{oo} – also call the guaranteed result in relative units, i.e. $\lambda_k(X^{oo})$ and according to the characteristic of the $f_k(X^{oo})$ TS it is impossible to improve, without worsening thus other characteristics.

Point of an optimum of X^{oo} and corresponding relative estimates of criteria $\lambda_1(X^{oo})$, $\lambda_2(X^{oo})$, $\lambda_3(X^{oo})$, $\lambda_4(X^{oo})$, are presented on direct to X^{oo} $\lambda_3(X^{oo})$ in fig. 3 ($\lambda_3(X^{oo})$, $\lambda_4(X^{oo})$ – are signed in fig. 3).

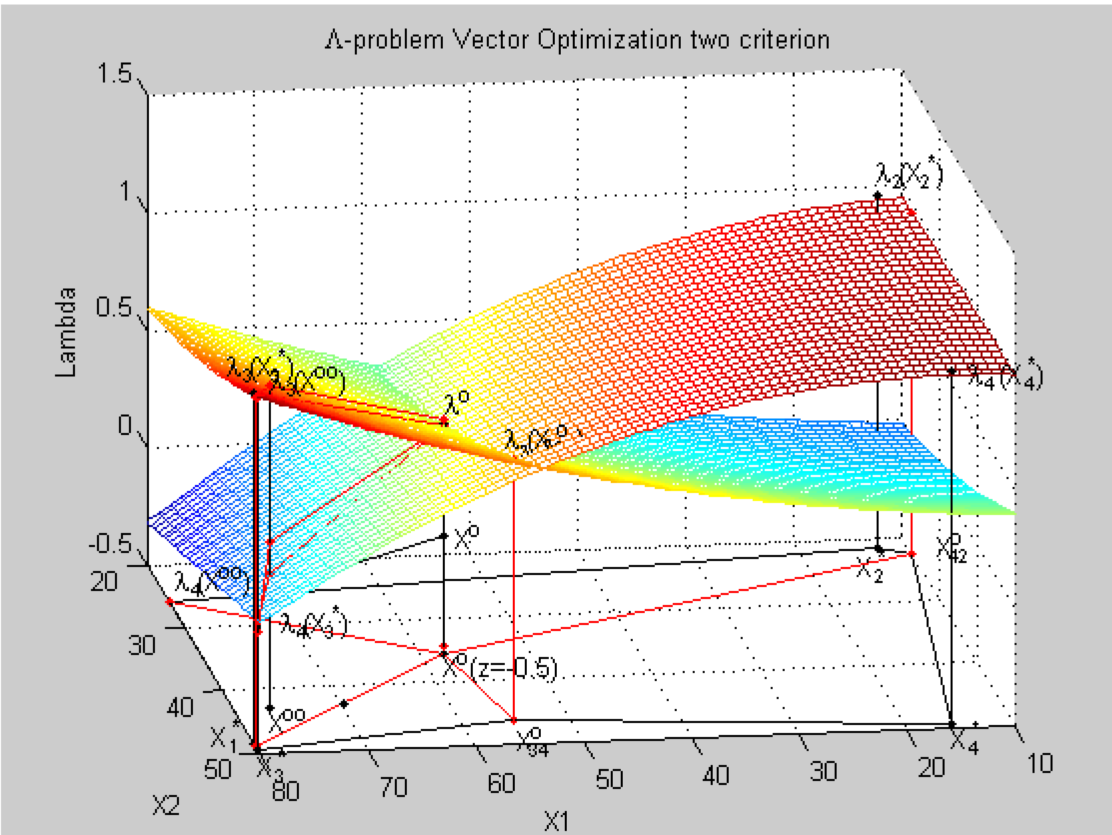

Fig. 3. Choice of the characteristic of the TS and its size in λ-problem with a priority of the third criterion

Step 8. Analysis of results.

The calculated size f $f_q(X^{oo})$=781, $q=3\in\boldsymbol{K}$ is usually not equal to the set f_q =750. The error of a choice of f_q is defined by an error of linear approximation of the valid change of relative estimates of $\lambda_k(X)$, $k=\overline{1,K}$ and a vector of priorities of

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$p_k^q(X)$, $k=\overline{1,K}$, $q\in K$ executed on steps 5, 6.

If error $\Delta f_q=|f_q(X^{oo})$ - $f_q|=|781\text{-}750|=31$, measured in physical units or as a percentage % $\Delta f_{q\%}=\Delta f_q$ / f_q *100, is less than set Δf , we pass to a step 10 if $\Delta f_q\geq\Delta f$, is carried out the following step.

The step 9-11 is carried out as required.

Step 12. End.

Conclusions

Thus, in work the methodology of optimization of parameters of difficult technical system on some set of functional characteristics that is one of the most important tasks of the system analysis and design is offered. The technology of creation of mathematical model of such system is presented in the form of a vector task and adoption of the optimum decision. For the solution of this task the methods based on normalization of criteria and the principle of the guaranteed result are used. Results of the decision are a basis for decision-making on the studied technical system. This methodology has system character - it can be used at research, modeling and adoption of the optimum decision for a wide class not only for technical systems of various branches, but also for systems economic, etc.

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